PROBLEM 1 - DZONGKHA

ཐཡཱ་འཕྲིན་དང་པོ། - རྫོང་ཁ

Problem 1 (20 points). Here are some numbers in Dzongkha and their numerical values:

1 — **ci** 3 — **sum** 8 — **ge** 12 — **cuɲi** 17 — **cupdyn** 19 — **cygu**

For higher numbers, Dzongkha uses two different systems (referred to here as A and B). Below are some numbers written in both systems, as well as their numerical values:

System A	System B	Value
ke ci da ɲi	**tsaɲi**	22
ke ci da ŋa	**tsaŋa**	25
ke pɟe-da ɲi	**sumcu**	30
ke ci da cyʑi	**soʑi**	34
ke ɲi da dyn	**ʑedyn**	47

System A	System B	Value
ke ko-da sum	**ŋaŋa**	55
ke sum da cuɖu	**døndu**	76
ke ʑi	**gepcu**	80
ke ʑi da gu	**ɟagu**	89
ke ceŋa	**sumɟa**	300

Finally, some equalities are given with left-hand side written in system A and right-hand side written in system B. Some numbers are missing.

	System A		System B
(1)	**cusum + ke pɟe-da ʑi**	=	**ɟasum**
(2)	**ɲiɕu ɲi**	=	**ɲiɕu × ʑipcu**
(3)	**ɲiɕu ci da ke sum da gu**	=	**(ŋapcu × gu) + cygu**
(4)	**ɲiɕu pɟe-da ɲi + ke pɟe-da ɖu**	=	**ŋapɟa + ɲiɟa cutãm**
(5)	**(ɲi × ko) + pɟe**	=	**ɲi**
(6)	**(ɲiɕu ko-da sum × pɟe) + ke pɟe-da sum**	=	**ɖukɟa**
(7)	**ɲiɕu ci da ke cuɖu da cuɖu**	=	**(ɟaʑi × ʑi) + ʑipɟa**
(8)	**ɲi × ɲiɕu ci da ke cutãm da gu**	=	**(____X × ɲiɕu) + copge**
(9)	**____Y + ke ci da ʑi**	=	**ɟaɖu**
(10)	**____Z + ke ko-da ɖu**	=	**dynɟa + sumɟa**

(a) Fill in the blanks X–Z with Dzongkha numbers.

(b) Write with digits the equalities (1–10).

(c) Write in Dzongkha in both systems: 75; 570.

⚠ The Dzongkha language belongs to the Sino-Tibetan family. It is spoken by approx. 171,000 people in Bhutan.

The words are given in a simplified transcription. **ɖ**, **ɟ**, **ɲ**, **ŋ**, **ɕ** and **ʑ** are consonants. **ã**, **ø** and **y** are vowels.

—Vlad A. Neacşu


CHINA
BHUTAN
INDIA
BANGLADESH
Punakha
Thimphu
Ha
Paro
Wangdü Phodrang
Tongsa
Bumthang
Lhuntsi
Tashigang
Mongar
Kanglung
Chima Kothi
Chirang
Shemgang
Chhukha
Dagana
Domphu
Pema Gatsel
Sibsoo
Phuntsholing
Gaylegphug
Deothang
Samdrup Jongkhar
Samchi
Shemgangkha
0 30 60 mi
0 40 80 km
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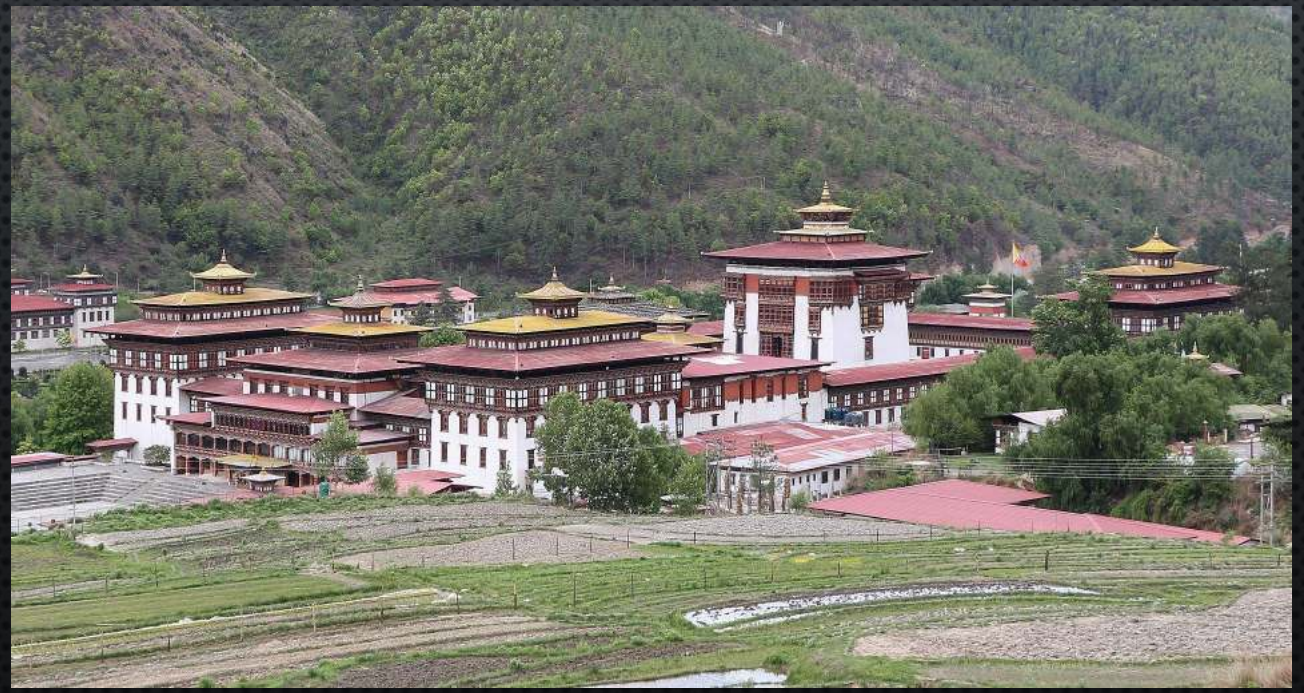
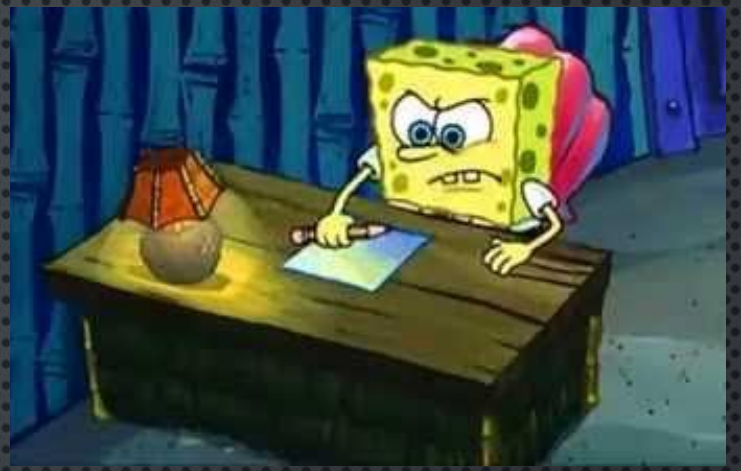
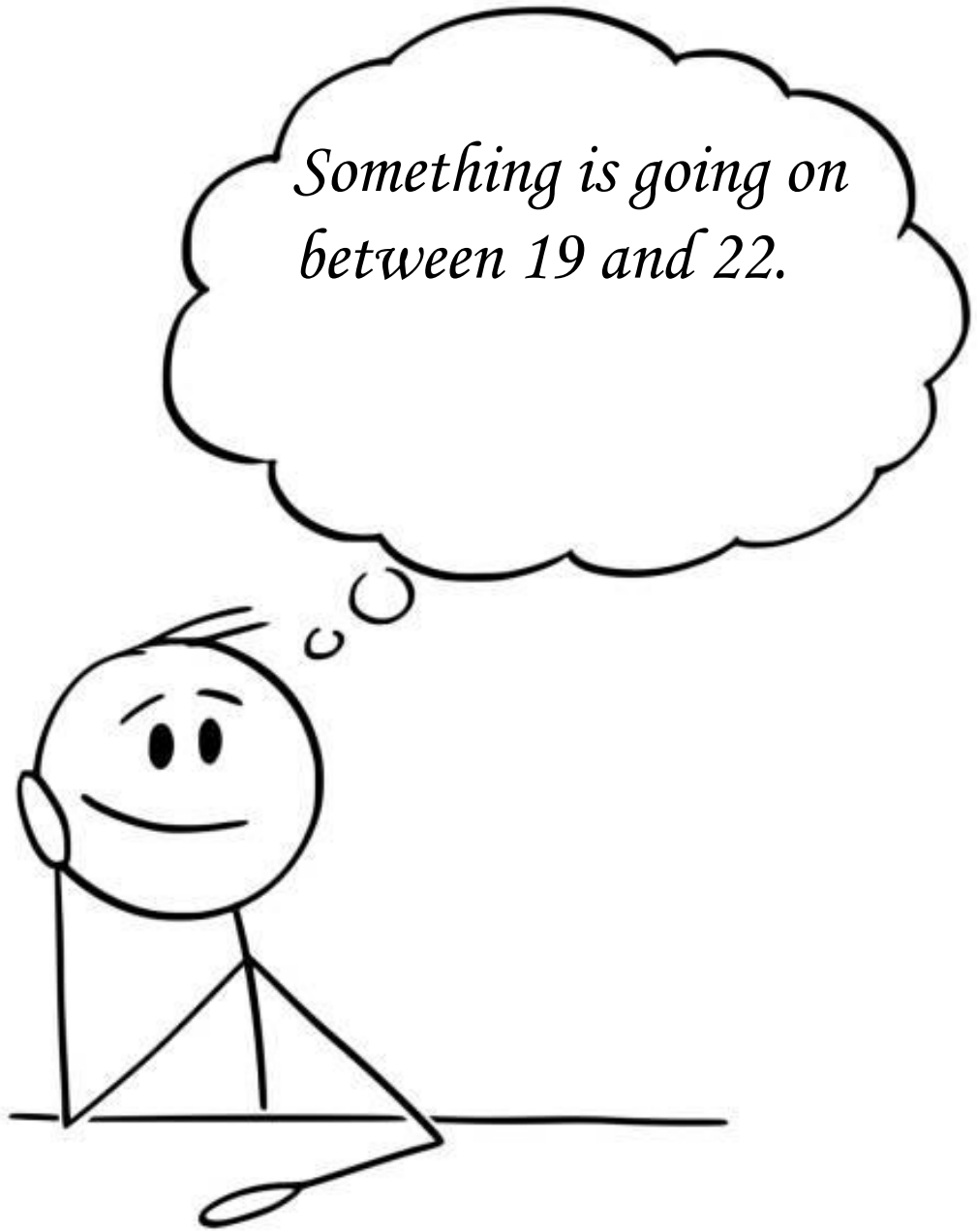
Khampa Tibetan
Dzongkha
Kurtöp
Dzala
Brokkat
Cho-ca-nga-ca-kha
Lakha
Bumthang
Dakpa
'Nyenkha
Chali
Brokpa
'Olekha
Kheng
Tshangla
Gongduk
Lepcha
Lhokpu
Nepali

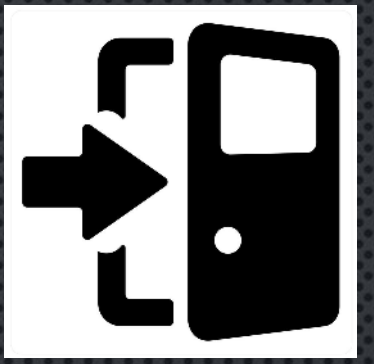


GRADING TEAM

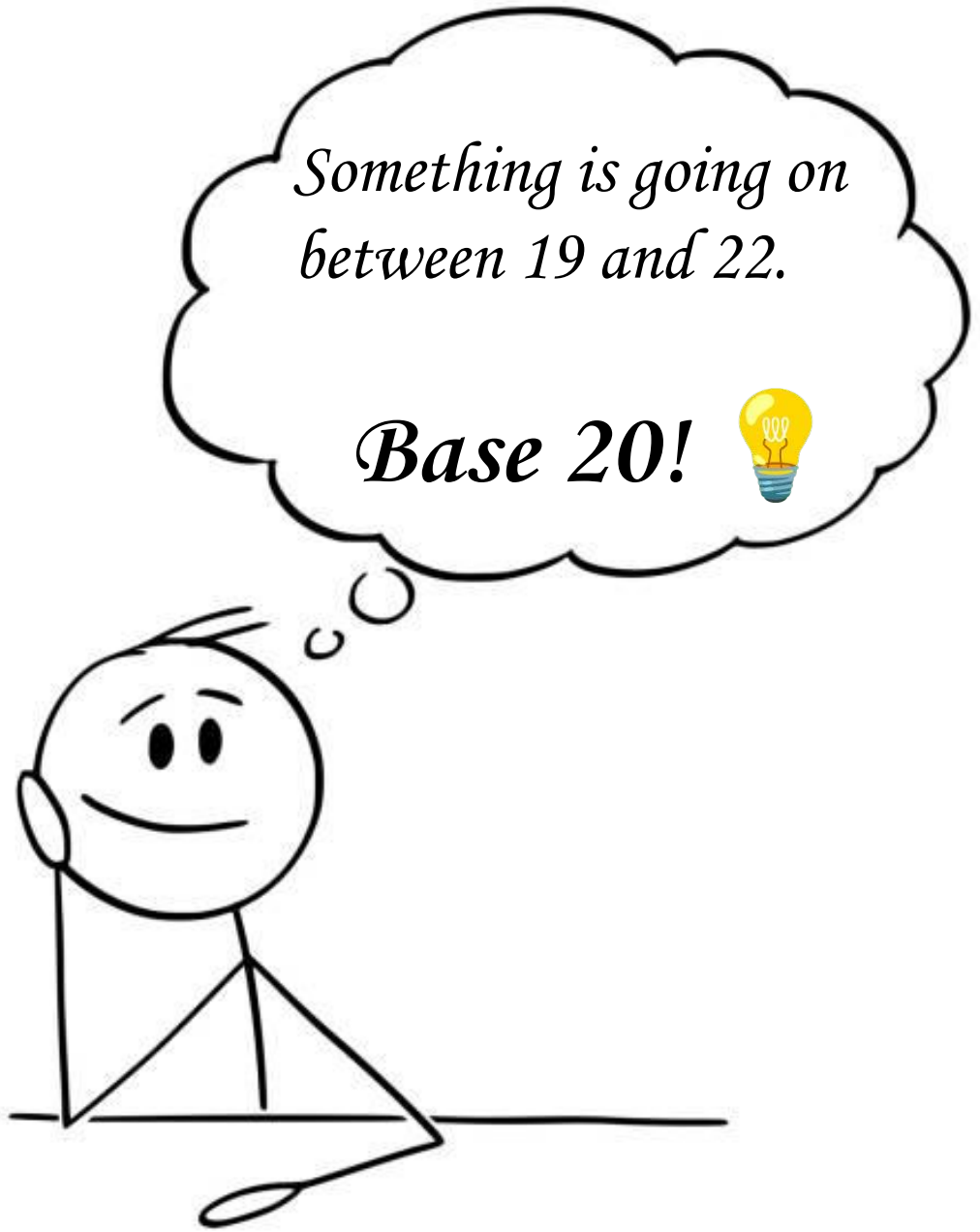


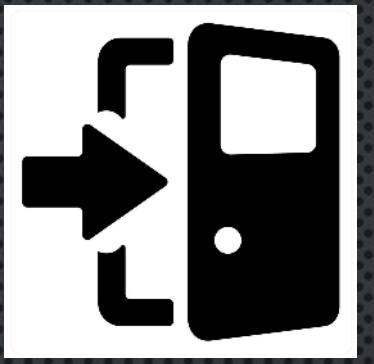
Milena Veneva
Stanislav Gurevich
Pan Tung-Le





19 = **cygu** (in both systems)
22 = **ke ci da ɲi** (system A)
tsaɲi (system B)





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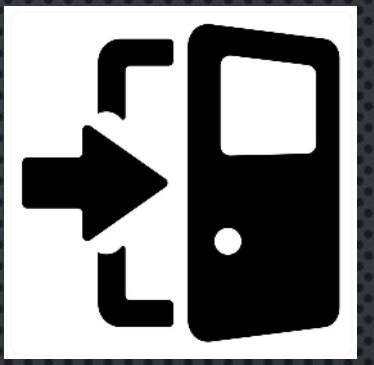
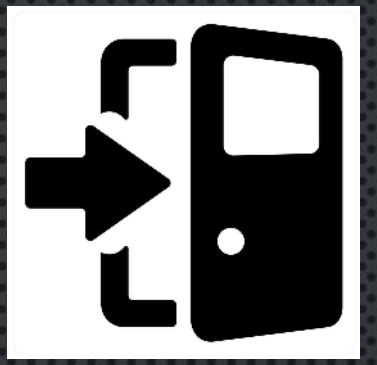
22 = **ke ci da ɲi** (system A)
tsaɲi (system B)



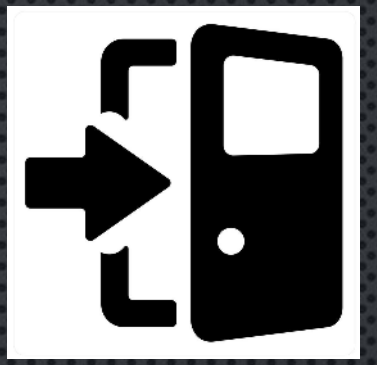
System A:
ke X da Y



System A:
ke X da Y

ke ci(1) da Y: 22, 25, 34
ke ji da Y: 47, 55
ke sum(3) da Y: 76

System A (base 20)
ke X da Y = 20X + Y



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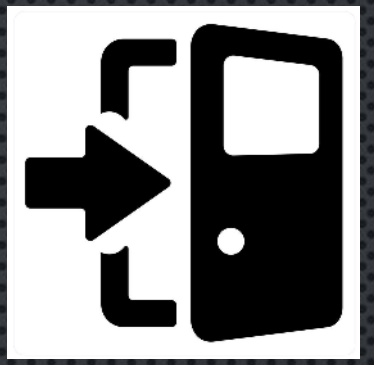
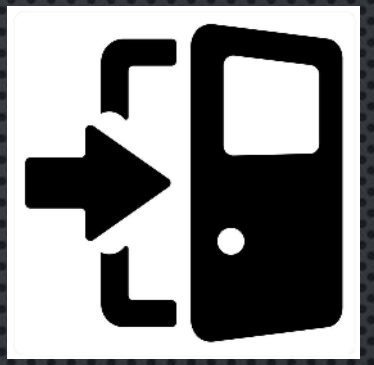


System A:
ke X da Y

ke ci(1) da Y: 22, 25, 34
ke ɲi da Y: 47, 55
ke sum(3) da Y: 76

System B:
ŋaŋa = 55
tsaŋa = 25
tsaɲi = 22

System A (base 20)
ke X da Y = 20X + Y



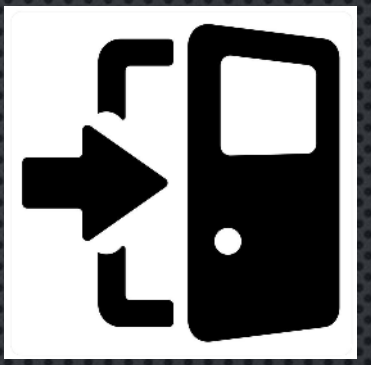
System A:
ke X da Y

ke ci(1) da Y: 22, 25, 34
ke ɲi da Y: 47, 55
ke sum(3) da Y: 76

System B:
ŋaŋa = 55
tsaŋa = 25
tsaɲi = 22

System A (base 20)
ke X da Y = 20X + Y

System B (base 10)
XY = 10X + Y



System A:
ke X da Y

ke ci(1) da Y: 22, 25, 34
ke ɲi da Y: 47, 55
ke sum(3) da Y: 76

System B:
ŋaŋa = 55
tsaŋa = 25
tsaɲi = 22

SYSTEM B (BASE 10)

ŋa-ŋa = 55
tsa-ŋa = 25
tsa-ɲi = 22

	10X	+Y
2	**tsa**	**ɲi**
3		
4		
5	**ŋa**	**ŋa**
6		
7		
8		
9		

SYSTEM B (BASE 10)

ŋa-ŋa = 55
tsa-ŋa = 25
tsa-ɲi = 22

so-ʑi = 34
ʑe-dyn = 47
døn-ɖu = 76
ɟa-gu = 89

	10X	+Y
2	tsa	ɲi
3		
4		
5	ŋa	ŋa
6		
7		
8		
9		

SYSTEM B (BASE 10)

ŋa-ŋa = 55
tsa-ŋa = 25
tsa-ɲi = 22

so-ʑi = 34
ʑe-dyn = 47
døn-ɖu = 76
ɟa-gu = 89

sumcu = 30
gepcu = 80
sumɟa = 300

	10X	+Y
2	tsa	ɲi
3	so	
4	ʑe	ʑi
5	ŋa	ŋa
6		ɖu
7	døn	dyn
8	ɟa	
9		gu

SYSTEM B (BASE 10)

ŋa-ŋa = 55
tsa-ŋa = 25
tsa-ɲi = 22

so-ʑi = 34
ʑe-dyn = 47
døn-ɖu = 76
ɟa-gu = 89

sumcu = 30
gepcu = 80
sumɟa = 300

	10X+	+Y	10X
2	tsa	ɲi	
3	so		sum
4	ʑe	ʑi	
5	ŋa	ŋa	
6		ɖu	
7	døn	dyn	
8	ɟa		gep
9		gu	

$$- \quad \beta \times 10 = (\beta)\text{-cu}, \ \beta > 2;$$
$$\beta \times 10 = (\beta)\text{-ɕu}, \ \beta = 2$$
$$- \quad \beta \times 10 + \alpha = ((\beta))\text{-}\boxed{\alpha}$$
$$- \quad \gamma \times 100\,(+\omega) = (\gamma)\text{-ʄa}\ \omega$$

	10X+	+Y	10X
2	tsa	ɲi	ɲi
3	so	sum	sum
4	ʑe	ʑi	ʑip
5	ŋa	ŋa	ŋap
6		ɖu	ɖuk
7	døn	dyn	dyn
8	ʄa	ge	gep
9		gu	

10X + Y = [10X+]—[+Y]

X = 2,9; Y = 1,9

10X = [10X]—**cu**

cu > ɕu for X = 2

X = 2,9

100X (+Y) = [10X]—ʄa (Y)

X = 2,9; Y = 1,99

SYSTEM B (BASE 10)

	10X+	+Y	10X	10+
2	**ʦa**	**ɲi**	**ɲi**	**cu**
3	**so**	**sum**	**sum**	**cu**
4	**ʑe**	**ʑi**	**ʑip**	**cy**
5	**ŋa**	**ŋa**	**ŋap**	**ce**
6		**ɖu**	**ɖuk**	**cu**
7	**døn**	**dyn**	**dyn**	**cup**
8	**ɟa**	**ge**	**gep**	**cop**
9		**gu**		**cy**

$10 + X = [10+]_X$—[+X]

10X + Y = [10X+]—[+Y] X = 2,9; Y = 1,9

10X = [10X]—**cu** X = 2,9

cu > ʁu for X = 2

100X (+Y) = [10X]—**ɟa** (Y) X = 2,9; Y = 1,99

SYSTEM A (BASE 20)

ke [+Y]$_X$ da [+Y]$_Y$ = 20X + Y

	10X+	+Y	10X	10+
2	tsa	ɲi	ɲi	cu
3	so	sum	sum	cu
4	ʑe	ʑi	ʑip	cy
5	ŋa	ŋa	ŋap	ce
6		ɖu	ɖuk	cu
7	døn	dyn	dyn	cup
8	ɟa	ge	gep	cop
9		gu		cy

System A	Value
ke ci da ɲi	22
ke ci da ŋa	25
ke pɟe-da ɲi	30
ke ci da cyʑi	34
ke ɲi da dyn	47

System A	Value
ke ko-da sum	55
ke sum da cuɖu	76
ke ʑi	80
ke ʑi da gu	89
ke ceŋa	300

SYSTEM A (BASE 20)

ke [+Y]$_X$ (da [+Y]$_Y$) = 20X (+ Y)

X = 1,19; Y = 1,19

	+Y
2	ɲi
3	sum
4	ʑi
5	ŋa
6	ɖu
7	dyn
8	ge
9	gu

System A	Value
ke ci da ɲi	22
ke ci da ŋa	25
ke pɟe-da ɲi	30
ke ci da cyʑi	34
ke ɲi da dyn	47

System A	Value
ke ko-da sum	55
ke sum da cuɖu	76
ke ʑi	80
ke ʑi da gu	89
ke ceŋa	300

SYSTEM A (BASE 20)

ke $[+Y]_X$ (da $[+Y]_Y$) = 20X (+ Y)

X = 1,19; Y = 1,19

ke pɟe-da X = ???

ke ko-da X = ???

	+Y
2	ɲi
3	sum
4	ʑi
5	ŋa
6	ɖu
7	dyn
8	ge
9	gu

System A	Value
ke ci da ɲi	22
ke ci da ŋa	25
ke pɟe-da ɲi	30
ke ci da cyʑi	34
ke ɲi da dyn	47

System A	Value
ke ko-da sum	55
ke sum da cuɖu	76
ke ʑi	80
ke ʑi da gu	89
ke ceŋa	300

SYSTEM A (BASE 20)

ke $[+Y]_X$ (da $[+Y]_Y$) = 20X (+ Y)

X = 1,19; Y = 1,19

ke pɟe-da X = ???

ke ko-da X = ???

ɲiɕu $[+Y]_X$ (da $[+Y]_Y$) = 400X (+ Y)

X = 1,19; Y = 1,399

ɲiɕu pɟe-da X = ???

ɲiɕu ko-da X = ???

	+Y
2	**ɲi**
3	**sum**
4	**ʑi**
5	**ŋa**
6	**ɖu**
7	**dyn**
8	**ge**
9	**gu**

SYSTEM A (BASE 20)

ke pɟe-da [2] = 30
ke pɟe-da [3] = 50
ke pɟe-da [4] = 70
ke pɟe-da [6] = 110

ke pɟe-da X = 20X – 10

Below are some numbers written in both systems, as well as their numerical values:

System A	System B	Value
ke ci da ɲi	tsaɲi	22
ke ci da ŋa	tsaŋa	25
ke pɟe-da ɲi	sumcu	30
ke ci da cyʑi	soʑi	34
ke ɲi da dyn	ʑedyn	47

System A	System B	Value
ke ko-da sum	ŋaŋa	55
ke sum da cuɖu	døndu	76
ke ʑi	gepcu	80
ke ʑi da gu	ɟagu	89
ke ceŋa	sumɟa	300

Finally, some equalities are given with left-hand side written in system A and right-hand side written in system B. Some numbers are missing.

	System A		System B
(1)	[illegible] + ke pɟe-da ʑi	=	ɟasum
(2)	ɲiɕu ɲi	=	ɲiɕu × ʑipcu
(3)	ɲiɕu ci da ke sum da gu	=	(ŋapcu × gu) + cygu
(4)	[illegible] + ke pɟe-da ɖu	=	ŋapɟa + ɲiɟa cutãm
(5)	(ɲi × ko) + pɟe	=	ɲi
(6)	(ɲiɕu ko-da sum × pɟe) + ke pɟe-da sum	=	ɖukɟa

SYSTEM A (BASE 20)

ke ɲɟe-da X = 20X – 10

ke ko-da X = 20X – 5

1/2

1/4

ɲiʁu ɲɟe-da X = 400X – 200

ɲiʁu ko-da X = 400X – 100

$$(2 \times \text{ko}) + \text{ɲɟe} = 2$$

SYSTEM A (BASE 20)

ke ɲɟe-da X = 20(X-1) + 10

ke ko-da X = 20(X-1) + 15

½

¾

ɲiʁu ɲɟe-da X = 400(X-1) + 200

ɲiʁu ko-da X = 400(X-1) + 300

$$(2 \times \tfrac{3}{4}) + \tfrac{1}{2} = 2$$

System A

- Base 20
- Overcounting for ½ and ¾
- **ɲiɕu = 400**

System B

- Base 10
- Five different forms for each digit
- **ɲiɕu = 20**

		1	2	3	4	5	6	7	8	9
$\boxed{k}$	k	ci	ɲi	sum	ʑi	ŋa	ɖu	dyn	ge	gu
	$10(+k)$	cu-	cu-	cu-	cy-	ce-	cu-	cup-	cop-	cy-
$\textcircled{k}$	$k \times 10$	cutãm	ɲi-	sum-	ʑip-	ŋap-	ɖuk-	dyn-	gep-	(gup-)
$\textcircled{\textcircled{k}}$	$k \times 10(+\ldots)$		tsa-	so-	ʑe-	ŋa-	(re-)	døn-	ɟa-	(go-)

- System A:
  - $\beta \times 20\ (+\alpha)$ = **ke** $\boxed{\beta}$ (**da** $\boxed{\alpha}$)
    - $\beta \times 20 + 10 \rightarrow$ **ke pɟe-da** $\boxed{\beta+1}$
    - $\beta \times 20 + 15 \rightarrow$ **ke ko-da** $\boxed{\beta+1}$
  - $\gamma \times 400\ (+\beta \times 20 + \alpha)$ = **ɲiɕu** $\boxed{\gamma}$ (**da ke** $\boxed{\beta}$ **da** $\boxed{\alpha}$)
    - $\gamma \times 400 + 200 \rightarrow$ **ɲiɕu pɟe-da** $\boxed{\gamma+1}$
    - $\gamma \times 400 + 300 \rightarrow$ **ɲiɕu ko-da** $\boxed{\gamma+1}$

- System B:
  - $\beta \times 10$ = $\textcircled{\beta}$**-cu**, $\beta > 2$;
    $\beta \times 10$ = $\textcircled{\beta}$**-ɕu**, $\beta = 2$
  - $\beta \times 10 + \alpha$ = $\textcircled{\textcircled{\beta}}$**-** $\boxed{\alpha}$
  - $\gamma \times 100\ (+\omega)$ = $\textcircled{\gamma}$**-ɟa** ω

(b)

(1) $13 + 70 = 83$

(2) $800 = 20 \times 40$

(3) $469 = 50 \times 9 + 19$

(4) $600 + 110 = 500 + 210$

(5) $2 \times \frac{3}{4} + \frac{1}{2} = 2$

(6) $1100 \times \frac{1}{2} + 50 = 600$

(7) $736 = 84 \times 4 + 400$

(8) $2 \times 609 = 60_X \times 20 + 18$

(9) $62_Y + 24 = 86$

(10) $885_Z + 115 = 700 + 300$

(a)

[X] = 60 = **ɖukcu**

[Y] = 62 = **ke sum da ɲi**

[Z] = 885 = **ɲiɕu ɲi da ke ʑi da ŋa**

(c)

- 75 = **ke ko-da ʑi** = **dønŋa**
- 570 = **ɲiɕu ci da ke pɟe-da gu** = **ŋapɟa dyncu**

TRIVIA

- System A (base 20) is used in the every day life, while System B (base 10) is more formal and it is calqued from Classical Tibetan.
- System A (base 20) is still widely used in counting amounts of houses, dogs, boxes, and crates.
- When stating an amount or a price, the decade is included in the number.

45 (**ʑeŋa**) will become **ʑipcu ʑeŋa** (40 45).

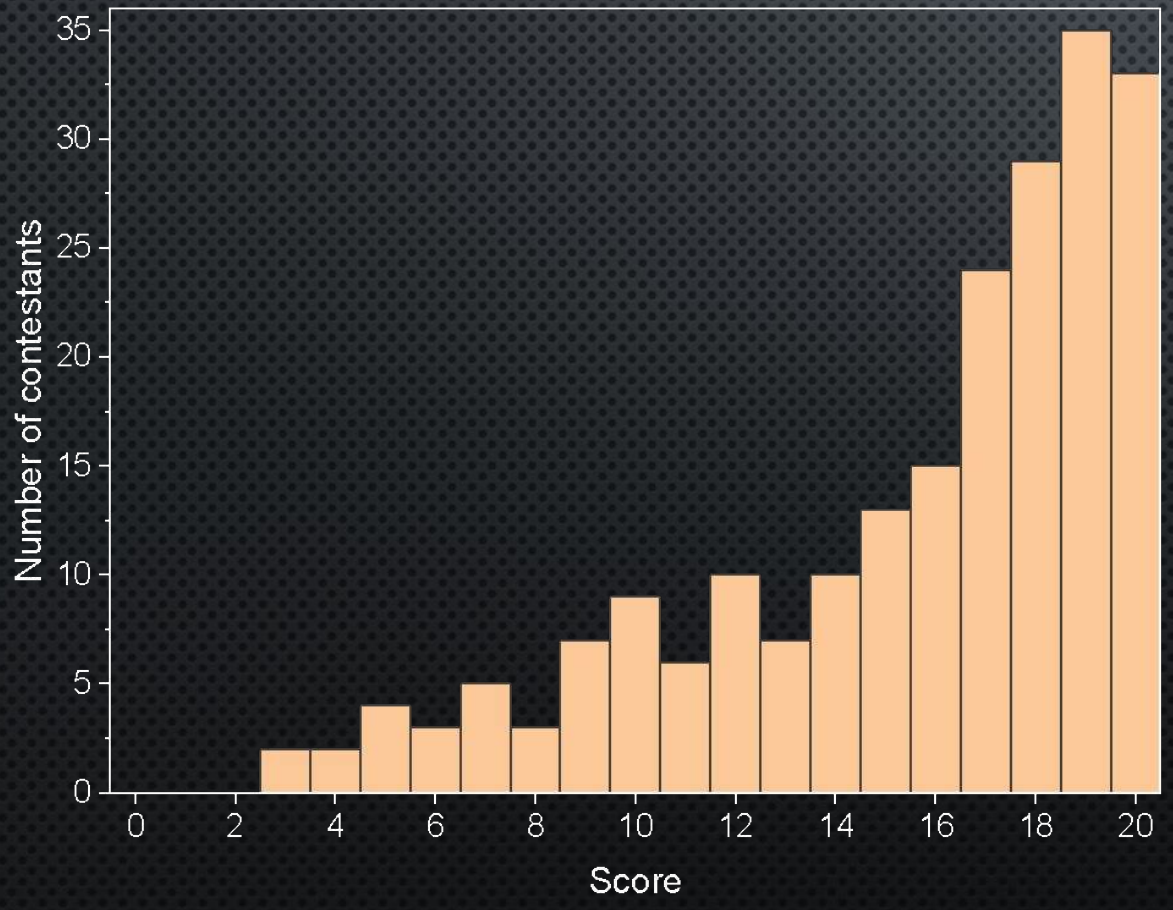
- Separate set of numerals for 21-29 used for the days of the months.

YOUR OPINION

EASIEST PROBLEM: 81/206 (39.3 %)

HARDEST PROBLEM: 11/206 (5.3 %)

OUR OPINION



SUBMISSIONS: 217/227 (95.6 %)

AVERAGE SCORE: 15.28 / 20

MAX SCORES: 11


THE
END?

BEST SOLUTION(S)

BEST SOLUTION(S)

Tananon Kinthorn

Thailand

BEST SOLUTIONS

BEST SOLUTIONS

Ekaterina Churkina

Belka